# Absolute Value

Most of you have seen absolute value before. You might know it as the two little lines that make everything positive. That's a good start, but there's more to the story of absolute value, which we'll explore in this worksheet. Start by simplifying a few expressions that involve absolute value:

1. $|3| =$
2. $|-5| =$
3. $|178| =$
4. $|0| =$
5. $|-95| =$

Hopefully that was easy. The hard part comes when we have something unknown inside the absolute value signs, like a variable, $x$. From the work above, can you come up with a rule that describes $|x|$ in words?

6. The absolute value of $x$ is ________ if $x$ is positive, and is ________ if $x$ is negative.
7. Let's modify this just a bit, since we've left out 0, which is neither positive nor negative. We can write it this way: The absolute value of $x$ is ________ if $x$ is nonnegative, and is ________ if $x$ is negative.

We can write this better using piecewise function notation, which looks like this. Write the definition without using any words at all, except for the "if" that's already there.

8.

$$|x| = \begin{cases} \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \end{cases}$$

What if it's not just $x$ in there? Simplify your answers as much as possible.

9.

$$|x+4| = \begin{cases} \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \end{cases}$$

10.

$$|3-2x| = \begin{cases} \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_\_ \end{cases}$$

So it seems part of the key to absolute value is the "if"s. That is, dividing it up into cases – "well, *if* this is true, then what happens, and *if* it's not true, then what? Can you solve an absolute value equation using this idea?

11. Solve the equation. That is, find all possible values for $x$ that make the equation true.

$$|x-4| = 12$$

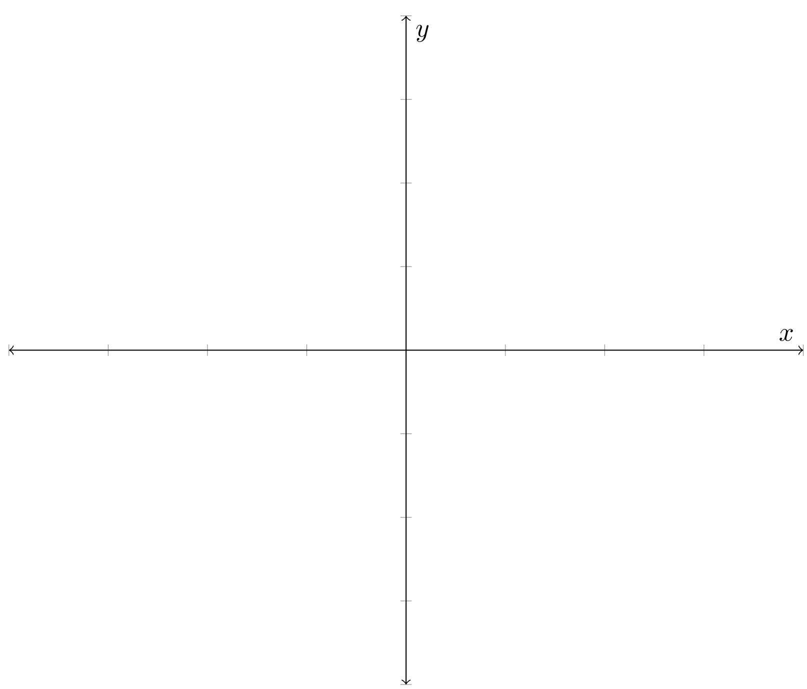
12. Solve the equation.
$$|4x + 1| = 3$$

13. Solve the equation.
$$|4x + 1| = 3x$$

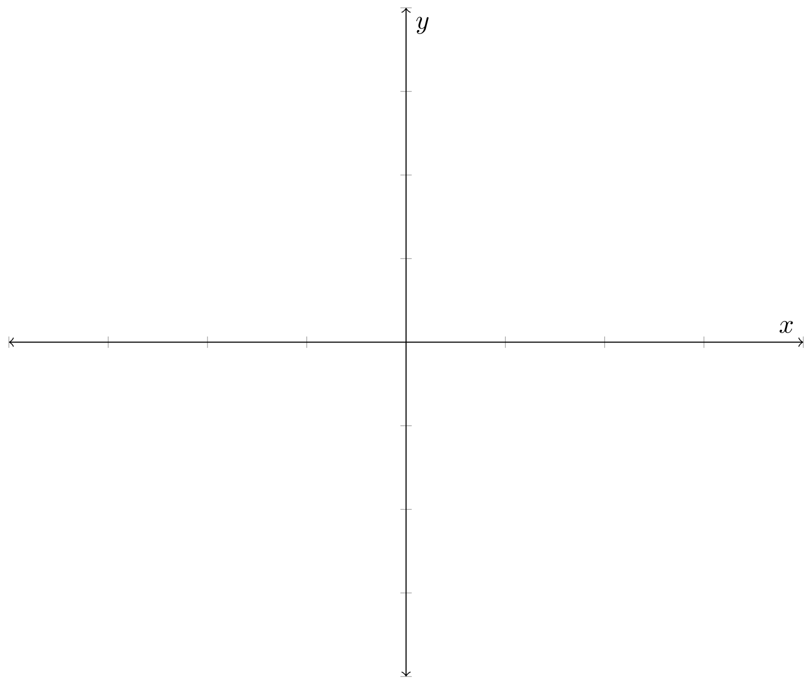
14. Solve the equation.
$$|2 - 2x| = 100$$

Let's take a look from the graphical perspective. First, we need to figure out how to graph a plain old absolute value function.

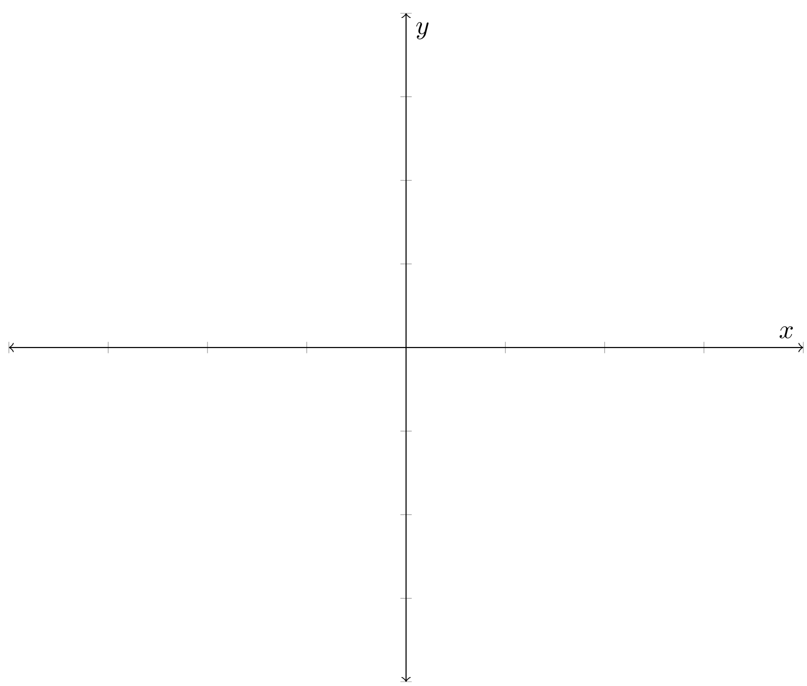
15. What would the graph of $f(x) = |x|$ look like? (Perhaps use the piecewise function notation to figure this out...it's really just two lines, right?)



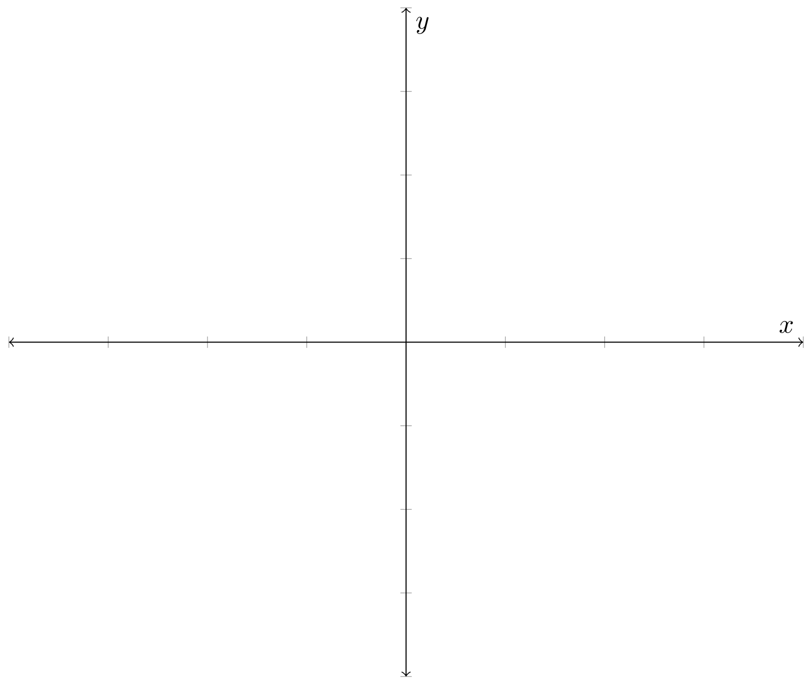
16. What about the graph of $g(x) = |3 - 2x|$? Can you also use the piecewise notation?




17. Now that you know how to graph absolute value functions, how can you interpret the equation $|4x + 1| = 3$ graphically? Draw a picture and describe in words.



18. What about $|4x + 1| = 3x$? Draw a picture and describe in words.




What about some inequalities?

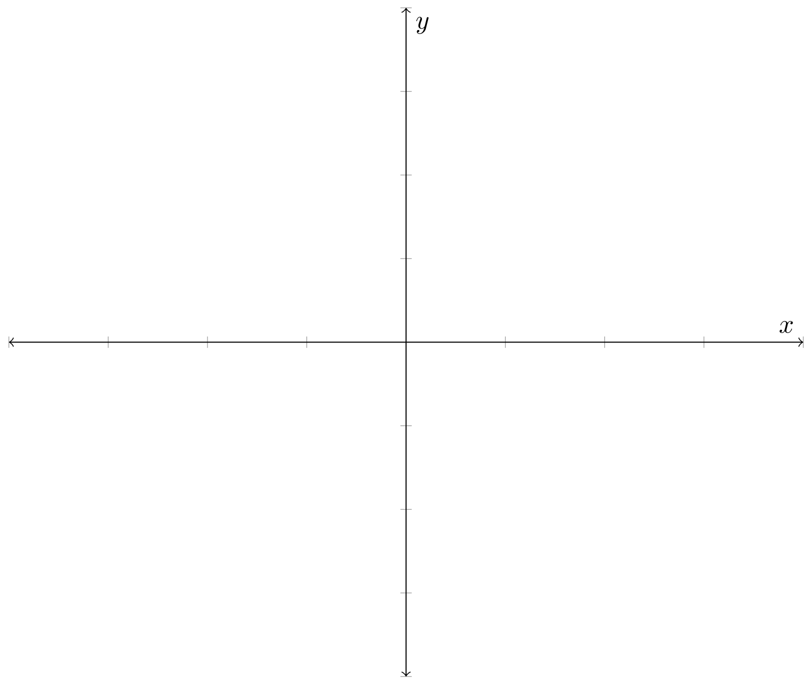
19. Solve the inequality. That is, find all possible values for $x$ that make the inequality true. Remember, divide things up into cases!

$$|x - 4| < 12$$

20. Can you interpret $|x - 4| < 12$ in words? Try to come up with the simplest description possible. It might help to draw the solution set on a number line first:

The solution set is all values of $x$ such that:

21. How would you interpret $|x - 4| < 12$ graphically?




22. Solve the inequality.

$$|4 - x| \geq 20$$

23. Solve the inequality.

$$|12x| < 4x$$

This might be a good time to see what the properties of absolute value might be. Things like the distributive law, factoring, adding absolute values, how do they work? Time to explore. Explain your reasoning!

24. True or False: $|x| + |y| = |x + y|$

25. True or False: $|x||y| = |xy|$

26. True or False: $|-x| = |x|$

27. True or False: $|2x + 2y| = 2|x + y|$

28. True or False: $|-3x - 6| = -3|x + 2|$

29. True or False: $|x^2| = |x|^2$

30. True or False: $|x + y|^2 = |x^2 + y^2|$

Now let's apply all that we've learned to some more complicated problems. There's no "right way" to solve these – but it's recommended you try breaking things down into cases. The hard part might be figuring out what cases to use, so let's start by just playing around a bit.

31. Take the inequality $|x + 1| + |x - 3| > 5$.
    (a) Is $x = -2$ a solution to this equation?

    (b) Is $x = -1$ a solution to this equation?

    (c) Fill out the table below, saying whether each value is a solution or not.

| Value | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| Solution? | | | | | | | |

    (d) What do you think would happen if you plugged in numbers larger than 4? Explain your answer.
        A. All the numbers would be solutions.
        B. None of the numbers would be solutions.
        C. Some would be solutions, and some wouldn't.
        D. No way to know without trying them all!

    Explanation:

(e) What about if you plugged in numbers less than $-2$? Explain your answer.

A. All the numbers would be solutions.

B. None of the numbers would be solutions.

C. Some would be solutions, and some wouldn't.

D. No way to know without trying them all!

Explanation:

(f) Is there a way to divide the entire number line, all real numbers, up into cases? Hint: three cases should be enough. (Remember that we have to consider all real numbers, not just integers.) Can you solve this problem completely?

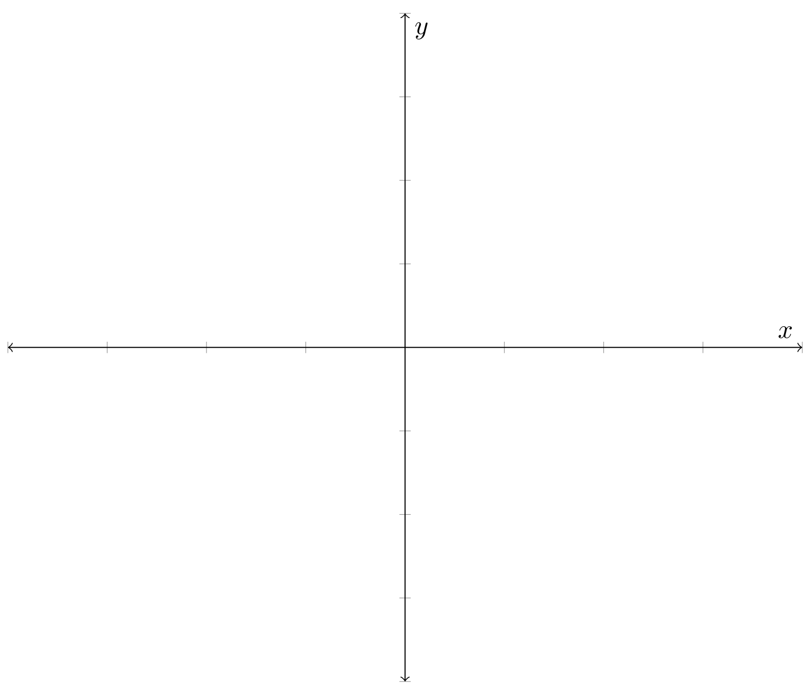
32. What about an equality? Can you use a similar approach? Try to solve:

$$|x-4|+|x-2|=3$$

33. What about something like this? (Hint: four cases needed.)

$$|x+3| = |x^2-1|$$

34. Could you interpret the previous questions graphically? The hard part is figuring out what the graph of $|x^2-1|$ looks like. How is the graph of $|x^2-1|$ related to the graph of $x^2-1$?




Let's note here that this idea of cases is not a new thing. You've been doing it ever since you started solving quadratic equations. You know, when you have something like $x^2-3x-4=0$, and you factor it to get $(x-4)(x+1)=0$, and then you say $x-4=0$ or $x+1=0$. That's dividing it up into cases, using the property that if two things multiply to make zero, one of them must be zero. That is, if $(x-4)$ times $(x+1)$ equals 0, then either $x-4$ equals zero (Case 1) or $x+1=0$ (Case 2). So you solve each of those cases, and the union of them forms your solution.

35. Are there any other times in math or elsewhere where you've divided things into cases to solve?